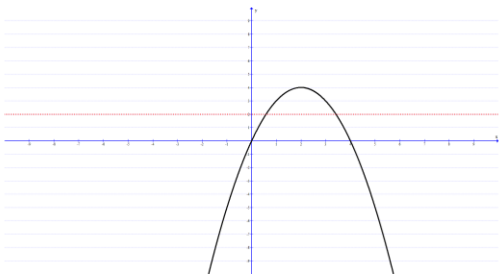
Section 6.2 solutions

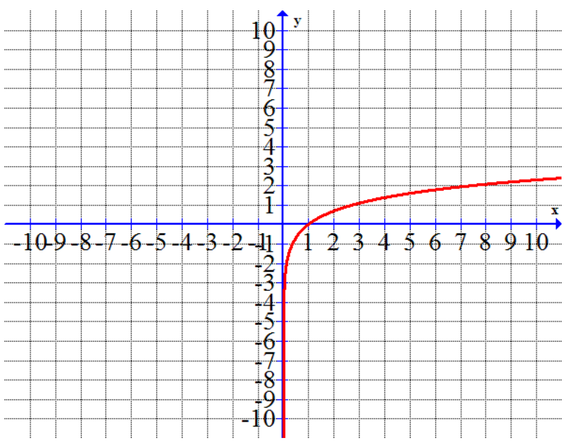
#1 - 4: Determine if the functions are one to one by using the horizontal line test.

1) This is NOT a one to one function as a horizontal line can be drawn to touch the graph in more than one place

Answer: not one to one



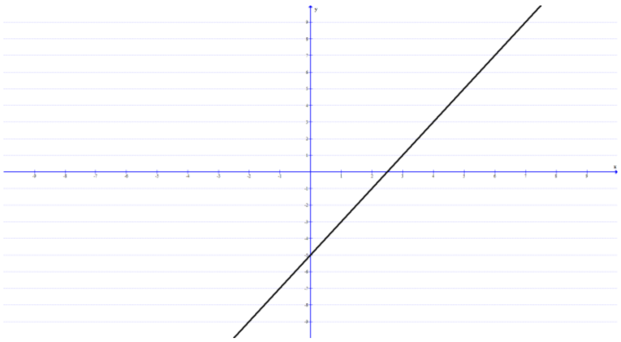
3) This is supposed to be a one to one function as no horizontal line can be drawn to touch the graph in more than one place.



Answer: one to one

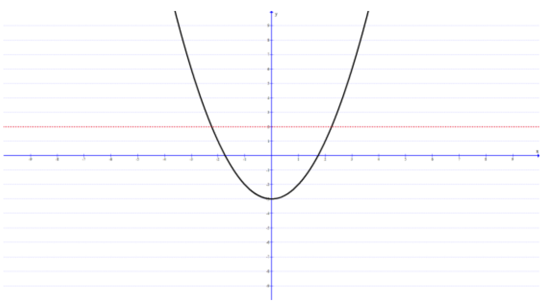
5) $f(x) = 2x - 5$

Answer: is one to one (graph passes horizontal line test)



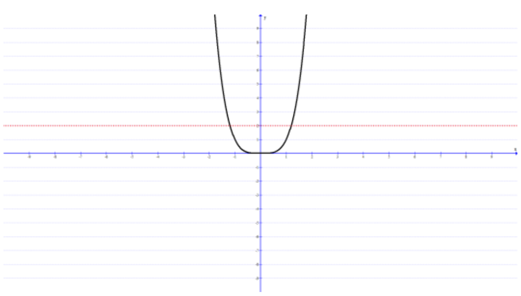
7) $f(x) = x^2 - 3$

Answer: not one to one (fails the horizontal line test)



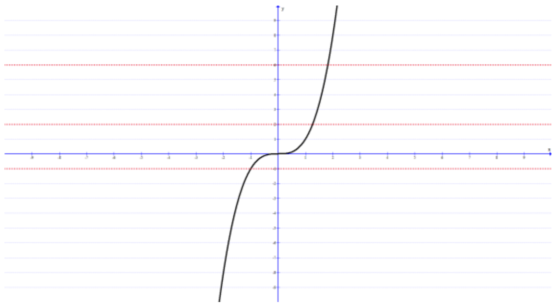
9) $g(x) = x^4$

Answer: not one to one (fails the horizontal line test)



11) $f(x) = x^3$

Answer: is one to one (graph passes horizontal line test)



#13 - 18: Determine which of the functions are one to one. If a function is one to one find its inverse.

13) f = { (0,1) (1,4) (2,4) (3,5)}
Answer: not one to one (duplicate y-coordinates)

15) h = {(0,3) (5,1) (7,11) (9, -3)}

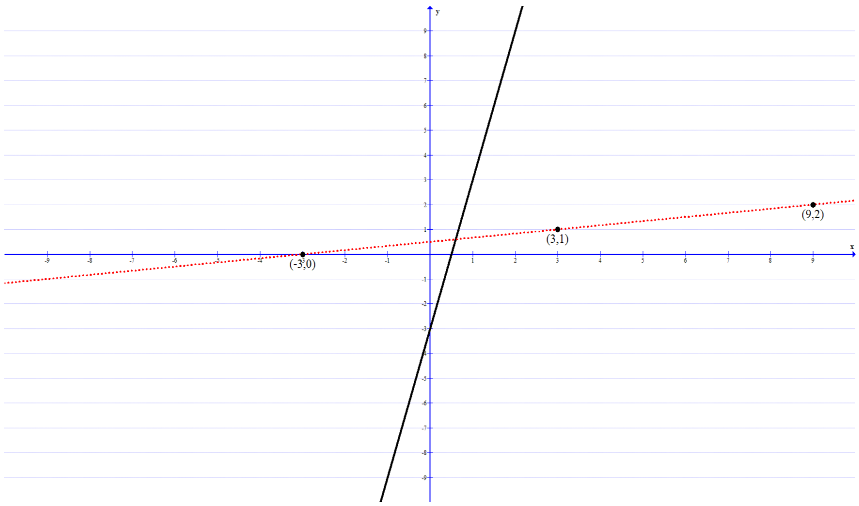
Answer: is one to one (all y's are unique)

We need to find the inverse since the function is one to one. Just switch the x and y's to do this.
h^{-1} = { (3,0) (1,5) (11,7) (-3,9)

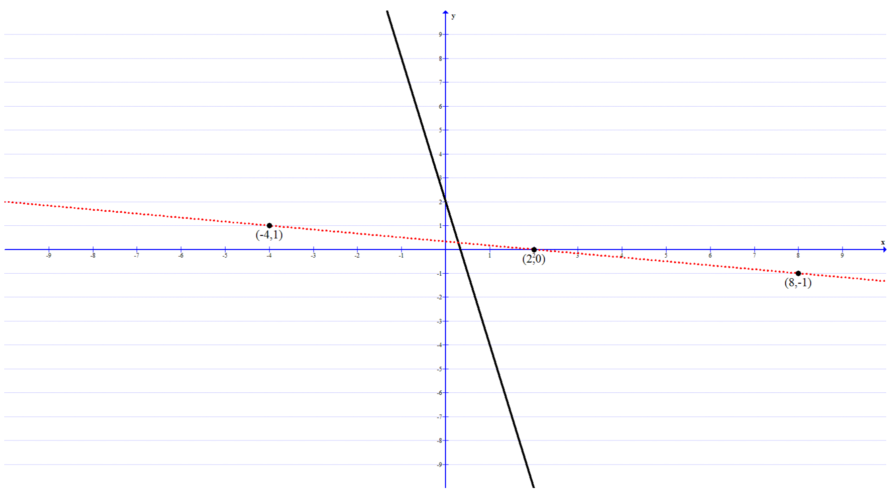
17) m = { (0,2) (2,3) (3, 5)}
Answer: is one to one (all y's are unique)

We need to find the inverse since the function is one to one. Just switch the x and y's to do this.
m^{-1} = { (2,0) (3,2) (5,3) }

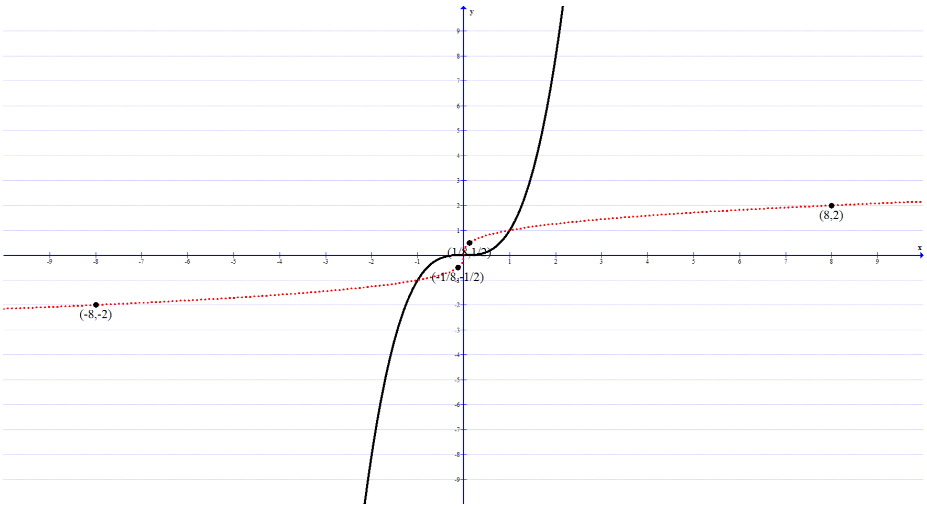
19) Switch the x and y of each given point, plot new points and draw graph. The new graph should have points (-3,0) (3,1) and (9,2)



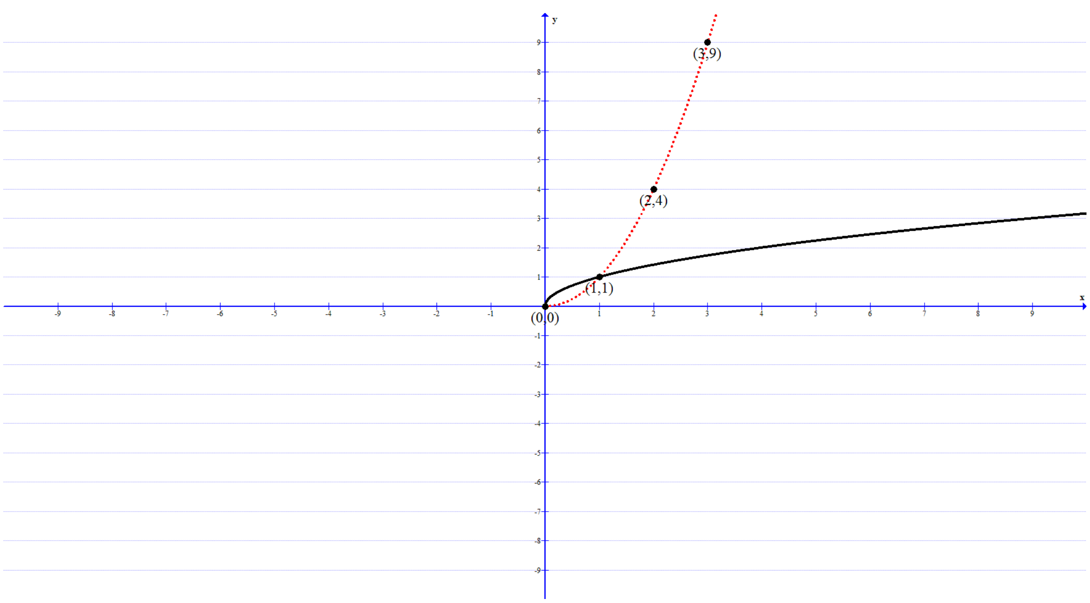
21) Switch the x and y of each given point, plot new points and draw graph. The new graph should have points (8,-1) (2,0) (-4,1)



23) Switch the x and y of each given point, plot new points and draw graph. The new graph should have points (1/8, ½) (-1/8, -1/2) (8,2) (-8,-2)



25) Switch the x and y of each given point, plot new points and draw graph. The new graph should have points (0,0) (2,4) (1,1) (3,9)



27) f(x) = 2x − 4

27a) Find the inverse of each function, and express it using appropriate notation.

First change function notation to y

y = 2x - 4

Second switch the x and y to create the inverse

x = 2y - 4

Third solve for y

x + 4 = 2y

$\frac{x+4}{2} = y$ (this fraction can be rewritten as $\frac{1}{2}x + 1 = y$)

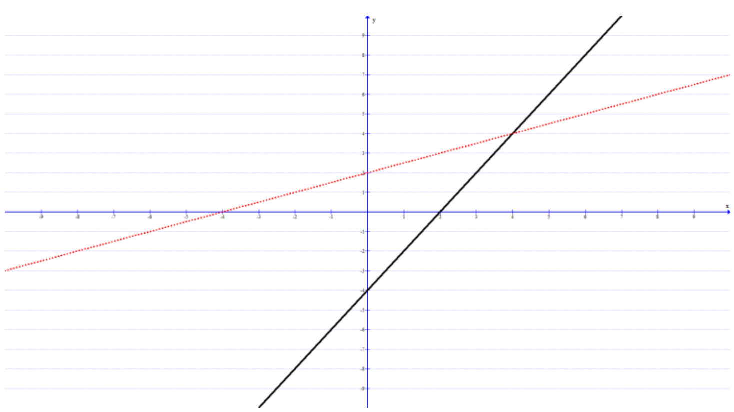
Fourth write answer with inverse notation

Answer: $\boldsymbol{f^{-1}(x) = \frac{x+4}{2}}$

27b) Check your answer by showing that $(f \circ f^{-1})(x)$ = x and $(f^{1} \circ f)(x)$ = x

$(f \circ f^{-1})(x) = f(f^{-1}(x))$ $(f \circ f^{-1})(x) = 2(f^{-1}(x)) - 4$ $(f \circ f^{-1})(x) = 2\left(\frac{x+4}{2}\right) - 4$ = x+ 4 – 4 = x	$(f^{-1} \circ f)(x) = f^{-1}(f(x))$ $(f^{-1} \circ f)(x) = \frac{f(x)+4}{2}$ $(f^{-1} \circ f)(x) = \frac{2x-4+4}{2} = \frac{2x}{2}$ = x

27c) Graph the function and its inverse and the line y = x on the same coordinate axis. (inverse drawn with dashed line)



29) $f(x) = \frac{x-2}{3}$

29a) Find the inverse of each function, and express it using appropriate notation.

First change function notation to y

$y = \frac{x-2}{3}$

Second switch the x and y to create the inverse

$x = \frac{y-2}{3}$ (multiply by 3 to clear fraction)

Third solve for y

3x = y – 2

3x + 2 = y

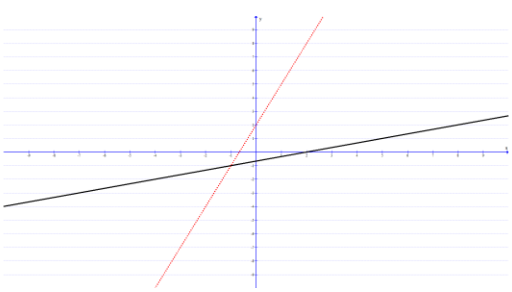
Fourth write answer with inverse notation

Answer: $f^{-1}(x) = 3x+2$

29b) Check your answer by showing that $(f \circ f^{-1})(x)$ = x and $(f^{1} \circ f)(x)$ = x

$(f \circ f^{-1})(x) = f(f^{-1}(x))$ $(f \circ f^{-1})(x) = \frac{f^{-1}(x)-2}{3}$ $(f \circ f^{-1})(x) = \frac{3x+2-2}{3} = \frac{3x}{3} = x$	$(f^{-1} \circ f)(x) = f^{-1}(f(x))$ $(f^{-1} \circ f)(x) = 3(f(x)) + 2$ $(f^{-1} \circ f)(x) = 3\frac{x-2}{3} + 2 = x-2+2 = x$

29c) Graph the function and its inverse and the line y = x on the same coordinate axis. . (inverse drawn with dashed line)



31) $f(x) = \frac{2}{x}$

31a) Find the inverse of each function, and express it using appropriate notation.

First change function notation to y

$y = \frac{2}{x}$

Second switch the x and y to create the inverse

$x = \frac{2}{y}$ (multiply by y to clear fraction)

Third solve for y

xy = 2

$y = \frac{2}{x}$

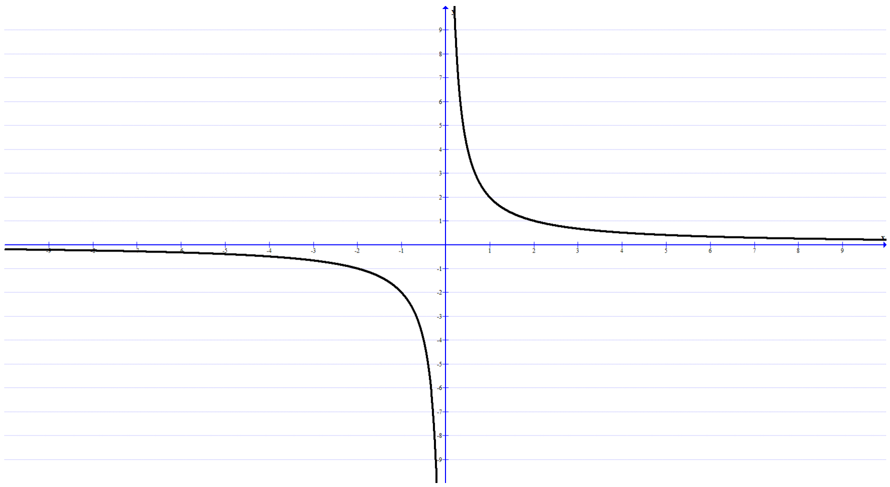
Fourth write answer with inverse notation

Answer: $\boldsymbol{f^{-1}(x) = \frac{2}{x}}$

31b) Check your answer by showing that $(f \circ f^{-1})(x)$ = x and $(f^{1} \circ f)(x)$ = x

$(f \circ f^{-1})(x)$ =$f(f^{-1}(x))$	$(f^{-1} \circ f)(x)$ = $f^{-1}(f(x))$
$(f \circ f^{-1})(x)$ =$\frac{2}{f^{-1}(x)}$	$(f^{-1} \circ f)(x)$ =$\frac{2}{f(x)}$
$(f \circ f^{-1})(x) = \frac{2}{\frac{2}{x}}= 2 * \frac{x}{2}$ =x	$(f^{-1} \circ f)(x)$ = $\frac{2}{\frac{2}{x}}= 2 * \frac{x}{2}$ =x

31c) Graph the function and its inverse and the line y = x on the same coordinate axis. . (only one graph shown as function is its own inverse)



33) $f(x) = \sqrt[3]{x-4}$

33a) Find the inverse of each function, and express it using appropriate notation.

First change function notation to y

$y = \sqrt[3]{x-4}$

Second switch the x and y to create the inverse

$x = \sqrt[3]{y-4}$

Third solve for y

$x^3 = \sqrt[3]{y-4}^{\,3}$

$x^3 = y - 4$

$x^3 + 4 = y$

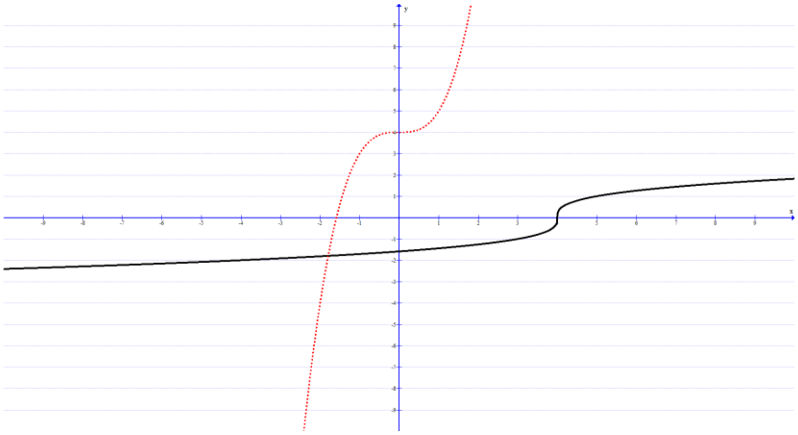
Fourth write answer with inverse notation

Answer: $f^{-1}(x) = x^3 + 4$

33b) Check your answer by showing that $(f \circ f^{-1})(x)$ = x and $(f^{1} \circ f)(x)$ = x

$(f \circ f^{-1})(x)$ $= f(f^{-1}(x))$ $(f \circ f^{-1})(x)$ $= \sqrt[3]{f^{-1}(x) - 4}$ $(f \circ f^{-1})(x)$ $= \sqrt[3]{x^3 + 4 - 4} = \sqrt[3]{x^3} = x$	$(f^{-1} \circ f)(x)$ $= f^{-1}(f(x))$ $(f^{-1} \circ f)(x)$ $= (f(x))^3 + 4$ $(f^{-1} \circ f)(x)$ $= \sqrt[3]{x-4}^{\,3} + 4 = x - 4 + 4 = x$

33c) Graph the function and its inverse and the line y = x on the same coordinate axis. . (inverse drawn with dashed line)



35) $f(x) = x^3 + 2$

35a) Find the inverse of each function, and express it using appropriate notation.

First change function notation to y

$y = x^3 + 2$

Second switch the x and y to create the inverse

$x = y^3 + 2$

Third solve for y

$x - 2 = y^3$

$\sqrt[3]{x-2} = \sqrt[3]{y^3}$

$\sqrt[3]{x-2} = y$

Fourth write answer with inverse notation

Answer: $\boldsymbol{f^{-1}(x) = \sqrt[3]{x-2}}$

35b) Check your answer by showing that $(f \circ f^{-1})(x)$ = x and $(f^1 \circ f)(x)$ = x

$(f \circ f^{-1})(x)$ =$f(f^{-1}(x))$	$(f^{-1} \circ f)(x)$ = $f^{-1}(f(x))$
$(f \circ f^{-1})(x)$ = $(f^{-1}(x))^3 + 2$	$(f^{-1} \circ f)(x)$ = $\sqrt[3]{f(x)} - 2$
$(f \circ f^{-1})(x)$ = $\sqrt[3]{x-2}^3 + 2$ = x-2 + 2 = x	$(f^{-1} \circ f)(x)$ = $\sqrt[3]{x^3 + 2 - 2}$ = $\sqrt[3]{x^3}$ = x

35c) Graph the function and its inverse and the line y = x on the same coordinate axis. . (inverse drawn with dashed line)